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The History of the Mixed Poynting Vector

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Abstract

This discussion of the importance of the mixed Poynting vector is given by way of a remark on the paper of H.-D. Geiler, K. Hehl, and D. Stock in 1983 [39]. It can be pointed out that the onset of this discussion can be traced back to the mid 19th century and to Stokes, v. Helmholtz, Lord Rayleigh etc. The mixed Poynting vector appears in all cases of reflection of electromagnetic waves at a boundary between two arbitrary media with different complex refractive indices. The energy flux density in the first medium supporting the incoming radiation is changed by the superposition of the incoming and the reflected wave. In the case where the first medium is absorptive, the interference effects lead to a specific energy saving. This changed energy flux is especially important in the case of laser application because the energy quantities are sufficiently great to be detected and/or to influence the technical or experimental applicability of laser power. In a short recapitulation of the main problems of understanding of the physical situation it is pointed out that this basic phenomenon has already been clarified in corresponding publications from 1950.

1. Introduction

Some time ago H.D. Geiler, K. Kehl, and D. Stock pointed out [39], that for the case of the energy absorption in a thin film, the mixed Poynting vector has to be considered if the film contains an incident and a reflected electromagnetic wave which produce interference phenomena by their superposition. This statement is true. However, the authors are wrong in characterizing their statement as a reference to a "new effect", hitherto neglected.

It is a fact that this is a very old problem with a very old history. The number of publications on this problem has become very great over the years [6] to [39] so that the impression could be gained that the problem is still open. However, this problem was solved many years ago [11] [12]. This paper is presented to give a recapitulation of the history of the mixed Poynting vector.

2. The History

In 1911, by now 75 years ago, Born and Ladenburg [7] called attention to the circumstances that in the case of a electromagnetic wave penetrating from a absorbing medium

into a nonabsorbing medium there appears a third energy flux, precisely the "mixed Poynting vector" which the cited authors believed to have discovered. However, in the paper of Born and Ladenburg the reality in physics of this mixed flux vector was judged to be doubtful because the authors stated a restricted interference ability for both waves, the incoming and the reflected wave. This line of reasoning was adopted to "preserve" the symmetry of transparency at the boundary, which was believed to be a primary principle.

The results of Born and Ladenburg were reflected in a monographical article by Koenig [8]. In this review article, the whole situation is explained in a manner giving the impression that Maxwell's Theory would fail in this case with respect to the principle of the conservation of energy. Naturally this is not at all the case.

In 1950, following a suggestion by Clemens Schaefer, the author again took up this problem and the reality in physics of the mixed energy flux vector was confirmed. It was clarified that the principle of energy conservation is valid and that the mixed Poynting vector plays the role of a "distribution vector". It is permissible to remark that the results of this investigation were presented in Bonn at the 1950 autumn meeting of the Deutsche Physikalische Gesellschaft. In the following discussion M. v. Laue agreed to the presented results. Some time before M. v. Laue had already published his own paper [6] as a contribution to the solution of the problem under discussion. In a similar fashion M. Born expressed his agreement in a personal talk with the author. After this it seemed that further questions about the mixed Poynting vector would be finished once and for all.

Nevertheless in the following years, new papers were continually presented on a new discovery and confirming this effect. In other papers the effect was wilfully (or even curiously) dismissed through argument.

It was the general importance of this controversial discussion with respect to the famous question of reciprocity of light beam propagation which has led to widespread interest. Vasicek in particular [22] to [30] [36] has dealt with this problem very intensively but without corresponding success. His formulae are quite different from all the normal expressions to be found as a consequence of Maxwell's theory in conventional textbooks on thin film optics. The main starting point of Vasicek's argument is the idea that the reversibility of a light beam and the symmetry of the energy transmission must be fulfilled independent of the light propagation direction. In a somewhat modified form, Keussler and coworkers also agreed with this primary assumption [21] [32] [34]. They based their idea on a "theorem" which was pointed out by H. v. Helmholtz [2], providing precisely this general formulation of the reciprocity. Helmholtz has included the case that absorptive media are also involved. However, the formulation of Helmholtz contains an additional expression which somewhat relativizes the statement, inasmuch as he stated: "as far as I can see".

In the older literature on optics [1] to [5] we also find some references to the reversibility of light beams within the context of analogous questions regarding the velocity reversal of mechanical motions. However, in such discussions it must be emphasized that all these statements are valid only for those cases where energy dissipation is absent. In the field of optics, this is synonymous with the absence of absorption.

Indeed, this is the decisive point in all thoughts on the question of reciprocity. If absorbing media are involved, one can expect in general an asymmetric transmission behaviour. In the case that only one of the two adjoining media is absorptive, the asymmetry is very easy to understand: if the medium of the incoming beam is non-absorptive, there is energy dissipation only in one wave, the transmitted wave. However, in the reverse direction of light propagation there is absorption in both the incoming and the reflected wave after they are superposed to form a resulting wave. This resulting wave contains a standing wave component due to interference. It seems clear that the resulting energy loss with respect to the transmitted beam is now quite different. There is therefore no symmetry.

We have a nonreversibility of light beam intensities not only in the case of a single boundary between two infinitely extended media but also in the case of light beams crossing a pile of different media between the infinite medium of the incoming beam and the other infinite medium carrying the outgoing beam. This pile of different media may also contain inhomogeneous regions. The angle of incidence to the stratification is without importance. Each medium may be absorptive or nonabsorptive. It was shown [11] [33] that full reversibility in transmission and reflection does not exist even if both terminating media have the same complex refractive index. However, it is easy to show [33], that the transmission and reflection are equal in both light beam directions if no absorption exists in the pile.

3. Discussion

If we now accept that the Helmholtz theorem is not valid where absorptive materials are involved then this does not contradict the law of energy conservation, as indicated by Vasicek and Keussler. The contrary is true, as only consideration of the mixed Poynting vector and the asymmetry of the light propagation leads to the energy conservation. In the energy balance equation (see for instance [38]):

$$\Phi_i + \Phi_r + \Phi_{i,r} = \Phi_t = \frac{1}{2} |E_t|^2 \frac{n_2}{Z_0} \quad (1)$$

Φ_i = incident radiation flux density

Φ_r = reflected radiation flux density

$\Phi_{i,r}$ = mixed Poynting vector

Φ_t = transmitted radiation flux density

$Z_0 = \sqrt{\mu_0/\epsilon_0} \approx$ vacuum wave resistor

the incident light beam is carried by an absorptive medium. In this case the mixed Poynting vector $\Phi_{i,r}$ is different from zero and the incident radiation flux density is not only given by Φ_i but additionally by $\Phi_{i,r}$. For the case that $\Phi_{i,r}$ is positive, the enhancement of the radiation flux density is a consequence of the reduced absorption in a thin "slice" in the medium "1" in front of the boundary where we have the standing wave due to interference. In regions of reduced resulting electric field strength, the energy loss is reduced and in consequence the radiation flux density passing into the next medium is

greater than it would be without the superposition between the incident and the reflected waves.

Vasicek and Keussler subtracted the value $\Phi_{i,r}$ on both sides of Eq. (1) without any further discussion, purely with the intention of restoring the symmetry in the energy flux which they felt lacking. In doing this, they ignore that the transmitted radiation flux density Φ_t is given by the expression $|E_t|^2 * n_2 / (2 * Z_0)$ and is therefore completely described by values which are characteristic only for the medium "2". The addition of a term which contains only values of the medium "1" must consequently falsify the result.

Through lengthy correspondence and supported by many publications [11] to [16] [25] [26] [31] [33] [35] [38] the author has been able to contradict Vasicek regarding the inadequacy of his calculations. There have also been other physicists [17] to [19] [24] [27] who strongly disapproved of Vasicek's misconception.

However, the argument about the mixed Poynting vector continues. In 1983 A. K. S. Thakur presented a paper [37] in which, due to his ignorance of all the preceding discussions, he treated the old problem in an inadmissible fashion. The author together with F. R. Keßler suggested an amendment [38]. The reader will find a corresponding discussion on the existence and the importance of the mixed Poynting vector in an article about the optical phenomena of light propagation through semiconductors containing great gradients in the complex refractive index [40].

4. Concluding Remarks

It currently seems that the application of laser radiation is leading to a revival of the old discussion because the differences in power occur at an order of magnitude where the interference effects which are established in the mixed Poynting vector are attaining greater importance with respect to the application of laser beam power and/or caloric effects. It would therefore appear understandable that publications are cropping up on the "new effect", which is, however, only a rediscovery. Thus it may be interesting to give here a historical review of the preceeding publications and discussions. At a time of separation of knowledge between different periodicals with respect to different subject this recapitulation of the fact that the problem has been solved since 1950 may also represent a contribution to international scientific communication. The compilation of the relevant literature may also be valuable to physicists who are interested in the history of his own branch of science.

It would be desirable that all the phenomena and quantitative calculations involved with the mixed Poynting vector find a final representation in a modern and competent textbook of optics, perhaps in context with Fresnel's formulae.

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